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OR MODELS FOR ENERGY PLANNING

Jeremy F. Shapiro

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MASSACHUSETTS
INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02139

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by

Jeremy F. Shapiro
Massachusetts Institute of Technology

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1. Introduction

Quantitative models for energy planning have proliferated recently, in large part as a result of the energy crisis and concern for the environment. Our intention here is to discuss some existing OR energy models and analyses, to present some ideas about how they can be extended, and to suggest where new models might be particularly useful.

It is somewhat arbitrary to try to identify distinct OR models and analyses within the class of quantitative energy models and analyses because there is a great deal of overlap between the various mathematical disciplines that are used. We have chosen to try to make this distinction, however, because we feel the emphasis thus far has been placed somewhat more heavily on the economic and econometric aspects of energy planning. By this we mean that greater time and effort has been spent on econometric estimation of energy supply and demand functions, models of interfuel substitution, economic consequences of energy

regulation and import duties, and so on. Less time and effort has been spent on the development and use of detailed mathematical programming models of energy systems, or the use of decision analysis and multi-criterion optimization methods in analyzing these models. Of course, econometric estimation and forecast of empirical relationships must precede the construction of large scale mathematical programming or other systems analysis models.

We will not attempt to provide a complete survey of OR energy models and analyses.[1] Instead we will discuss three types of OR energy planning models which we feel are the most important and the most illustrative. Included are energy system equilibrium models, R & D planning models for new energy technologies, and emergency oil stockpiling and allocation models. Moreover, the concern will be directed mainly at strategic planning models as opposed to tactical operating models. Mathematical programming models have long been used to solve tactical problems such as scheduling oil refineries, extending gas pipelines, and so on.

2. Energy System Equilibrium Analysis

The structure of supply and demand relations for energy commodities is naturally dynamic involving time lags in market response. A number of researchers have been concerned with combining econometric forecasts of supply and demand with mathematical programming models of oil and gas distribution, oil refinery

1. An excellent survey of all types of energy models is given by Charpentier (1974).

production, refinery and coal mining capacity expansion when steel and capital resources are limited, and so on. The implied dynamic optimization models, however, have been judged too difficult to construct and solve and therefore single period equilibrium models have been proposed.

The typical energy equilibrium model consists of two sets of commodities: desired or consumer commodities and primary or unproducible commodities.[1] There are three sets of variables: a vector of desired and primary commodities denoted by d, a vector of prices for these commodities denoted by p, and a vector of production and distribution of commodities denoted by x. A simple form of equilibrium models in these variables is

$$p = Rd + r \tag{1a}$$

$$Ax = d \tag{1b}$$

$$c + p'A \geq 0 \tag{1c}$$

$$(c + p'A)x = 0 \tag{1d}$$

$$\begin{aligned} &x \geq 0, \\ &d_i \geq 0 \text{ if commodity } i \text{ is primary} \\ &d_i \leq 0 \text{ if commodity } i \text{ is desired} \end{aligned} \tag{1e}$$

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1. Other economic sectors, such as agriculture, are viewed as having a third set of intermediate commodities as well; see Hall et. al. (1975).

The constraints (1a) are the econometrically estimated relationships between supply, demand and prices. The constraints (1b) are the production and distribution (trade) constraints; the production activities taken as a whole are usually assumed to form a Leontief matrix. The constraints (1c) state that no production activity can make a profit in equilbrium because capital would otherwise move into the activity. The vector c includes capital as well as operating costs. The constraints (1d) state that only activities which breakeven can be operated at a positive level in a competitive equilibrium.

The model described above is precisely the world oil market model developed by Kennedy (1974) where the commodities are crude oil and four refined products. The matrix A pertains to the production of refined products from crude oil and the transportation of crude oil and refined products from one region of the world to another. Equilibrium models concerned with the geographic distribution of commodities are called spatial equilibrium models and were first studied by Samuelson (1952); see also Takayama and Judge (1971), Hall et. al. (1975).

Brooks (1975) has developed a spatial equilibrium model similar to (1) for the U.S. natural gas pipeline network. In Brooks' model, the matrix A in (1b) is the node arc incidence matrix of the U.S. gas pipeline network. The econometric relations

in (1a) are derived from the econometric models of MacAvoy and Pindyck (1974). Brooks' model has the additional feature that there are constraints on the prices due to regulation.

The FEA Project Independence Blueprint Study used an equilibrium model somewhat similar to (1) to integrate the various parts of the Study (Hogan (1974)). The FEA model considers all energy commodities and the econometric relationships are non-linear. In addition, they consider, at least implicitly, resource constraints on the production of energy commodities such as constraints on refinery capacity. In other words, the FEA model considers additional constraints of the form $Qx \leq q$ in problem (1).

There are other mathematical programming models which are quasi-equilibrium in character because they are static. Included in this category are the models of Debanné (1973) and the Brookhaven Energy System Optimization Model (BESOM) developed by Hoffman (1972). Both models can be viewed as network representations of energy systems with supply, demand and production nodes, and distribution arcs connecting the nodes. Given starting conditions on prices, demands, reserves, pipeline capacities, and so on, these models minimize the cost of satisfying demand, including pollution costs, for a generic or equilibrium period. The models are different in that Debanné is more concerned with production and distribution capacities of the system, and Hoffman is more concerned about interfuel substitution, particularly with respect to new technologies.

A common solution technique proposed for solving problem (1) is quadratic programming. This technique is suggested because the equilibrium conditions (1) can be interpreted (Kennedy (1974)) as the Kuhn Tucker optimality conditions for the quadratic programming problem

$$
\begin{aligned}
\max \quad & -r'd - \frac{1}{2}d'Rd - cx \\
\text{s.t.} \quad & Ax - d = 0 \\
& x \geq 0 \\
& d_i \geq 0 \quad \text{if commodity } i \text{ is primary} \\
& d_i \leq 0 \quad \text{if commodity } i \text{ is desired.}
\end{aligned}
\qquad (2)
$$

The objective function in (2) has been interpreted as consumer surplus or net social pay-off by Samuelson (1952). In the case of one commodity, it represents the area under the curve of the desired commodity versus price minus the sum of the area under the curve of primary commodity versus price and the cost of production and transportation. The matrix R may not be symmetric, or more generally, the objective function in (2) may not be concave, implying the Kuhn-Tucker conditions are necessary but may not be sufficient for optimality. Moreover, the convergence of quadratic programming algorithms (e.g., see chapter 24 in Dantzig 1963) is not guaranteed.

The simple linear form of the econometric relations (1a), or to a lesser extent, the linear production and transportation constraints (1b), may be unsatisfactory for some energy applications. Nonlinear economic equilibrium models, if they are not too large, can be solved by the simplicial algorithms for finding fixed points devised by Scarf; see Scarf and Hansen (1973). At present, these algorithms appear to have a practical limit of being able to solve equilibrium problems of 80 commodities or less, but recent innovations may permit larger problems to be solved. The number of commodities in the FEA model is several hundred; the number in Kennedy's model is 30.

Many of the economic equilibrium models discussed above are partial equilibrium models; that is, the interaction of the U.S. energy sector with other sectors of the economy is ignored and certain important variables like GNP are taken as exogenous. The pervasive influence of the energy sector on the U.S. and world economies makes partial analysis more difficult to accept than partial analyses of other economic sectors. An alternative approach has been offered by Hudson and Jorgenson (1974) which combines a complete econometric model of the U.S. with an input-output analysis of the U.S. energy sector; the input-output coefficients are endogeneous variables determined by the model.

With this background, we turn to a discussion of the possible extensions of equilibrium models and analyses using OR

methodology. An immediate analytical area requiring further study are the convergence properties of the various algorithmic procedures proposed for computing equilibria. A great deal of research has been done on complementary pivot theory algorithms for solving linear equations such as (1), including some results for problems where the underlying quadratic form is indefinite (e.g., Cottle and Dantzig (1968), Eaves (1971)). Moreover, the nature of these algorithms makes sensitivity analyses easy to perform.

Plessner (1967; p. 177) points out that "the existence of market equilibrium becomes very acute in applying the above model. It is very likely...that for some particular set of data problem (1) will have no solution." The absence of an equilibrium can be due to conflicting externalities. For example, constraints added to (1) imposing lower bounds on production and demand and upper bounds on pollution could result in an infeasible problem.

Alternatively, if R is not negative definite, there is the possibility of multiple equilibria. Experience with similar equilibrium models indicates that multiple equilibria can exist. For example, the Urban Institute model (DeLeeuw (1972)) for analyzing urban housing markets is designed to find prices for urban dwellings yielding a Pareto optimum among a number of households. It is known for this model (Ferreira (1975)) that multiple equilibria can exist; a unique equilibrium solution can in principle be selected by maximizing landlord's profit over the set of equilibria. The

difficulty is that this set can be disconnected and impossible to characterize completely.

The criterion of maximizing consumer surplus has a certain validity and is convenient because it produces linear econometric relations to be estimated. Nevertheless, there are other criteria which one can consider. Mancke (1974) suggests five criteria for U.S. energy policy evaluation: (1) economic efficiency, which could be measured by consumer surplus; (2) net U.S. supply (total U.S. petroleum demand minus total U.S. supply from all sources); (3) U.S. national security, particularly as it relates to stock-piling petroleum; (4) pollution control; (5) the magnitude and distribution of any petroleum and natural gas based rents. More generally, these criteria and perhaps others should be considered simultaneously in some type of multi-criterion analysis of U.S. energy equilibria models.

In discussing the use of Hoffman's model to assess new technologies, Cherniavsky (1974; p. 18) states, "A...relatively general set of objectives may be stated as the supply of energy at minimum cost, an adequate supply with minimal environmental consequences, and national self-sufficiency in energy resources. There are inherent contradictions between these goals, as for example between minimum cost and either minimal environmental impact or national self-sufficiency, that must be resolved in a planning activity."

An obvious approach to resolving the conflict of diverse optimality criteria is to analyze these energy planning models as multicriterion optimization problems. We will not attempt to survey the vast literature on multicriterion optimization; an excellent survey is given by Cohon and Marks (1975). Rather we will discuss briefly some consequences of taking this point of view with respect to energy equilibrium models. Assume for the moment that numerical functions $f_k(x,d,p)$ exist for the different criteria other than consumer surplus. The usual approach for generating efficient or Pareto optimal solutions to problem (2) is to solve (2) with the various terms weighted by positive coefficients. The Kuhn-Tucker condtions for this new problem do not yield an interpretable set of relations between supply, demand and price such as (1a) which can then be estimated. This anomaly requires further study.

It is unlikely, however, that the preferences for various energy policy criteria can be easily identified and quantified. Procedures exist for progressive articulartion of preferences to a decision or optimization problem; e.g. see Benayoun et. al. (1971). Some research in this direction has been done by Gros (1974) who used multi-attributed utility theory to describe preferences in nuclear reactor siting.

An equilibrium model such as (1) is a convenient and useful mathematical artifice permitting the study and steady state

solution of a dynamic process with stationary data. If the dynamic process satisfies certain conditions (e.g., the underlying mapping is a contraction mapping), its finite horizon solutions converge to a unique equilibrium solution. This permits the more easily computed equilibrium solution to be interpreted as the optimal immediate solution to be taken in the first period of a planning problem with a long, but finite planning horizon. Grinold (1974a) has a comprehensive discussion of these properties of dynamic programming models including dynamic Leontief systems.

The most serious drawback of the energy planning models discussed above is their inability to go beyond the equilibrium solutions and treat the dynamics of the U.S. and world energy sectors. For example, Hogan (1974; p. 23) states, "The approximations or limitations in the (FEA) study of dynamics are the most serious deficiencies in the conceptual framework. The determination of an equilibrium balance for a given set of conditions is viewed as a static problem without a time dimension...(There is no) guarantee that the time path implied by the 1977 equilibrium will be consistent with that of the 1980 equilibrium, for example." Thus, there is a distinct awareness among energy modelers that dynamic analyses are needed, but the state of the art of mathematical programming is judged to be insufficiently advanced to permit it. We make the optimistic assumption that methodological breakthroughs in dynamic optimization are close at hand and discuss some important related

research questions.[1]

First, there is the question of identifying the stationary dynamic process of which the spatial equilibrium model (1) represents the steady-state. Takayama and Judge (1971) discuss the question briefly but it appears to have received little attention. Let p_0 and d_0 be given prices and demand at the start of a planning horizon. A dynamic process implied by (1) is of the form

$$\begin{array}{lll} Ax_0 = d_0 & c + p_t'A \geq 0 & \\ & & t=1,2,\ldots \qquad (3) \\ p_{t-1} = Rd_t + r & (c + p_t'A)x_t = 0 & \qquad (3) \\ Ax_t = d_t & & \end{array}$$

Let $S(p_0,d_0)$ denote the set of $\{p_t,d_t,x_t\}$ satisfying (3). Let α be a discount factor and for any $\{p_t,d_t,x_t\}$, define

$$V(\{p_t,d_t,x_t\}) = \liminf_{T\to\infty} \sum_{t=0}^{T} \alpha^t(-r'd_t - \tfrac{1}{2}d_t'Rd_t - cx_t), \qquad (4)$$

which is a measure of discounted net social pay-off over the infinite horizon. The suggested optimization problem is

$$\begin{array}{l} \max V(\{p_t,d_t,x_t\}) \\ \text{s.t. } \{p_t,d_t,x_t\} \in S(p_0,d_0). \end{array} \qquad (5)$$

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1. An important exception is the allocation of energy resources model of Nordhaus (1973). Nordhaus overcame the implementation pitfalls by developing a highly aggregate model for which demand is exogeneously given and not responsive to price.

Grinold (1974b) has studied similar infinite horizon linear programming models for which he has developed finite horizon approximation problems, duality results and some equilibrium analysis. We mention that convergence of the sums in (4) depend on the discount factor α; this parameter is a critical factor in studying dynamic planning models and sensitivity analysis on it should be performed. The extension of Grinold's results to the quadratic programming spatial equilibrium model is an area of future research.

It is important to mention that there has recently been significant new research in approximation methods based on subgradient optimization (Held, Crowder and Wolfe (1974)) for large scale optimization models. Subgradient optimization is effectively a heuristic method for approximating the primal-dual simplex algorithm for these problems (Fisher, Northup and Shapiro (1974)). Grinold (1972) has shown how dynamic linear programming problems can be analyzed as primal-dual pairs by the primal-dual algorithm. Thus, a synthesis would appear possible for decomposing dynamic linear programs into a series of static models linked by a dual pricing mechanism that is optimized by subgradient optimization.

There is, of course, considerable doubt about the appropriateness of the stationarity assumptions required for equilibrium analysis and the dynamic extensions just mentioned. The U.S. and world energy sectors are clearly undergoing permanent shifts in

character away from cheap oil and natural gas. The timing, sizing and phasing of new energy technologies will be crucial, and careful planning is needed. The use of OR models and analyses in this regard is discussed in the following section.

3. R & D Planning Models for New Energy Technologies

For many years preceding the current energy crisis, it was recognized that energy technologies relying on exhaustible resources would have to be replaced ultimately by new technologies based on, for example, solar or nuclear energy. Even with the coming of the energy crisis, however, new technologies have been slow in developing, partly because of the great expense and long lead times involved. Another contributing factor to the delay is the great uncertainty about the extent of the world's reserves of exhaustible energy resources.[1] Whatever the extent, most energy planners accept the notion that substantial R & D in new technology is required immediately.

There are three types of new technologies to be identified and compared in energy planning: (1) exotic technologies, such as solar energy or fast breeder reactors, which do not yet exist; (2) expensive technologies, such as shale oil extraction, which are not yet economically competitive with existing technologies; and

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1. Kaufman (1975) has stated that there is 800% disagreement among experts on the quantity of proven oil and gas reserves in the U.S., and 1500% disagreement over the unproven reserves.

(3) controversial new technologies, such as nuclear generated electric power or strip mining, which are not yet acceptable for environmental reasons. One must study not only the effectiveness of these technologies independent of all others, but their competition in terms of interfuel substitution. This is a complicated process involving not only the penetration of new technologies into the energy market, but also changes in the mix of existing technologies as the result of changes in regulation, demand and price relationships, political factors, and scarcity rents reflecting the exhaustible character of petroleum, natural gas and coal.

Most energy planning models developed to date analyze new technologies in a macroeconomic or macroscopic fashion to estimate efficiency and cost parameters, and macroeconomic impact. The planning model which treats new technologies in the greatest detail is BESOM mentioned in the previous section (Cherniavsky (1974)). Programs separate from the linear programming model are used to compute efficiencies, environmental impacts and unit costs for new technologies. Base cases are established without the new technologies for future years such as 1985 or 2000, and the impact of new technologies are measured against them. Since the model is static, constraints are applied to limit the degree of substitution of new technologies in a given planning year. The shadow prices on each of these constraints indicates the profitability of increasing the pentration of the new technology. However, as we remarked in the previous section, the

critical dynamic and transient character of the energy market is not being modeled. Uncertainty in the timing of entry of the new technology is not treated. The model deals with uncertainty in cost and effectiveness by sensitivity analysis. Combined effects of groups of technologies are also studied.

The model of Nordhaus (1973) focuses on new technologies because he is concerned with "the efficient allocation of energy resources over time by determining the cheapest way of meeting a growth path of final demands (exogeneously determined) for energy products with a given stock of energy resources and a given set of processes for converting resources into products."[1] With respect to new technologies, his model indicates that "the optimal solution depends to a certain extent on unproven technologies. The system simply cannot run very long without development of a breeder reactor, fusion technology, or some other process that rests on a virtually inexhaustible resource base. But time is not particularly pressing, and the economy can wait at least 100 years for this ultimate technology. The need for other sorts of technology is more pressing. In particular, some form of synthetic liquid fuel must be developed quite rapidly to replace petroleum when the latter is exhausted. Such processes are in development - shale oil and coal liquefaction being the most significant - but they have not yet proved their economic and environmental acceptability."[2]

1. Nordhaus (1973), p. 566

2. Ibid., p. 568

Carter (1974) has developed and analyzed a dynamic input-output model to evaluate the effects of pollution abatement and new energy technologies on the rate of economic growth over the next 10 to 15 years. The closed dynamic input-output

$$(I - A)x - B\dot{x} = 0 \tag{6}$$

is used to make this evaluation, where x is a vector of economy-wide and energy variables and $\dot{x}$ is a vector of their time derivatives. The matrices A and B are current account and capital coefficient matrices, respectively. The model also contains a household component which corresponds to rows in A and B whose coefficients represent income and indirect tax payments; and columns in A and B which represent expenditures by households and government plus net exports. If all sectors were to grow at a uniform rate, equation (6) would imply

$$(I - A - \lambda B)x = 0 \tag{7}$$

where λ is the "turnpike" growth rate. Empirical evidence has indicated that the growth of the U.S. economy has been historically reasonably close to the computed path. In the study performed by Carter, the matrices A and B were 83-order. The effect of new technologies is measured against a base case with the 1970 coefficient matrices. The changes investigated included changes in electric power generation, transmission and distribution, increments in

fossil-fuel consumption, coal gasification, energy related capital stocks, and pollution controls. All of these changes are realized as changes in A and B thereby causing changes in λ, the balanced growth rate.

A number of quantitative R & D models have been developed by operations research analysts for private sector R & D decision making; e.g., Lockett and Gear (1973), Souder (1973). These models address to a much greater degree than the models discussed above the detailed decisions to be made in managing multi-stage R & D programs. A basic construct in many of these models is the project tree which is a stochastic decision tree consisting of decision nodes and chance nodes, and taking into account resource consumption and the values of possible end products. Uncertainty is allowed in project duration, resource requirements, project outcome and project value. Once a multi-stage R & D model is constructed, the implied mathematical programming problem is to allocate resources so as to optimally select the projects to be begun in the first period. This mathematical programming problem can be formulated as a stochastic linear or integer programming problem, possibly of large problem size because of the large number of branches on the project trees. Another approach is to combine simulation with the linear or integer programming problem, working backwards in time sampling optimal decisions at chance nodes in the project trees. This approach can become computationally excessive as well.

The greater use of these models in energy planning would be beneficial in a number of ways. Benefits would be derived from the process of rational specification of the models as well as the quantitative answers provided by them. For example, much would be learned from the subjective assessment by experts of the degree of technological advancement in solar energy required to make it a significant factor in the energy sector; or, similar assessment of the probability that the fast breeder reactor is a feasible technology by a certain date. Methods for statistical assessment of new military technological advancements are given in Press and Harman (1975).

In the private sector, the value of a successful R & D project can often be fairly well predicted in terms of penetration of a new product into a known market and the expected profits from such a penetration. For new energy technologies, the value must be measured in terms of its impact on the entire energy sector and the U.S. economy. In other words, the project tree and mathematical programming optimization for R & D management discussed in Lockett and Gear (1973) should be combined with the macroeconomic models of Hoffman, Carter, and others, to measure the project values. One can envision a decomposition approach which alternates between project selection and analysis, and macroeconomic evaluation of the outputs of the analysis. As discussed in the previous section, the value or cost of a new technology is probably multi-attributed implying the

need to assess the utility of these attributes to different parties and to optimize in a multi-criterion sense.

An important planning phenomena to be studied by quantitative R & D models would be government intervention to stimulate R & D activity in the private sector. The costs of energy R & D are so large, the uncertainties so great, and the payoff period so long that private industry has appeared unwilling or unable to launch major R & D programs. Government subsidies to private industry could be included as decision variables in the models, and the justification for such government investment could be measured by the economic returns in later periods.

An application of the stochastic decision tree and mathematical programming approach to new energy technology planning is given by Manne (1975). This model attempts to select an optimal mix of electricity generating plants, focusing on the uncertainty of the availability date of nuclear breeder reactors. The model does not, however, treat the R & D management problem for the breeder reactor. Hax and Wiig (1974) have done a private sector decision analysis study of R & D in shale oil extraction, including deciding whether or not to bid on a leasing of U.S. government property with shale oil fields.

4. Emergency Stockpiling and Allocation Plans

The emergency U.S. oil stockpiling problem has a great deal in common with traditional operations research models for inventory control and facilities location so as to minimize the sum of construction and distribution costs. Abandoned mines and depleted reservoirs are the lowest in cost, but they are not ideally located and have porous formations which can limit the rate of withdrawal. Steel storage tanks can be located anywhere, in some regions offshore, but they are much more expensive and steel resources are limited. Salt domes may be the most cost effective locations for storing large quantities of oil. Mixed integer programming models for this problem have been proposed by Copp and MacRae (1974).

The emergency oil stockpiling problem is unusual in that there is an important game theoretic aspect to the rate of buildup and ultimate size of the stockpiles. The U.S. is particularly vulnerable to embargo at the early stages of the buildup and less and less vulnerable as the stockpile builds up. This implies that the objective function for the emergency storage and distribution problem must include a complicated cost component measuring the expected cost of economic disruption as a function of stockpiled inventories.

If an embargo occurs, or oil is in short supply for other reasons such as insufficient production of certain refined products,

then the U.S. government is faced with the need to develop an oil allocation plan. Little work has been done to date on quantitative allocation models. Moore (1973) develops a regional linear programming for rationalizing the allocation of oil products and refining capacity for discretionary uses to different regions of the country. The objective function he suggests is to maximize output as a function of employment.

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